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## Research Article

# Model-Based Control Design of Series Resonant Converter Based on the Discrete Time Domain Modelling Approach for DC Wind Turbine

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This paper focuses on the modelling of the series resonant converter proposed as a DC/DC converter for DC wind turbines. The closed-loop control design based on the discrete time domain modelling technique for the converter (named SRC#) operated in continuous-conduction mode (CCM) is investigated. To facilitate dynamic analysis and design of control structure, the design process includes derivation of linearized state-space equations, design of closed-loop control structure, and design of gain scheduling controller. The analytical results of system are verified in z-domain by comparison of circuit simulator response (in PLECS™) to changes in pulse frequency and disturbances in input and output voltages and show a good agreement. Furthermore, the test results also give enough supporting arguments to proposed control design.

## 1. Introduction

MEDIUM-voltage DC (MVDC) collection of wind power is an attractive candidate to reduce overall losses and installation cost, especially within offshore HVDC-connected wind generation as illustrated in Figure 1 [1]. To connect DC wind turbine with MVDC network ( $\pm 50\text{kV}_{\text{DC}}$ ), the series resonant converter (SRC) serves as a step-up solid-state transformer as shown in Figure 2. With the series resonant converter, the DC turbine converter can take advantages of high efficiency, high voltage transformation ratio, and galvanic fault isolation for different ratings of turbine generator [2–6].

Traditional closed-loop control of SRC for the DC distribution system is easily implemented by detecting the zero-crossing of the resonant inductor current  $i_r$  and controlling the length of transistor and diode conduction angle  $\alpha$  without considering circuit parameters of SRC [7]. Additionally, the output power flow control of SRC for DC network is achieved by controlling the phase-shift angle and frequency between the two arms of H-bridge inverter [6, 8, 9].

Based on the discrete time domain modelling approach, the small-signal model of an improved SRC (named SRC#)

is proposed [9, 10]. This paper continues with the small-signal plant model addressed in Section 3 and the Appendix and mainly focuses on the closed-loop control design for the system. In the following sections, the mode of operation of SRC# and small-signal plant model based on the discrete time domain modelling approach will be briefly introduced first. The structure of closed-loop control based on the proposed small-signal plant model and the improvement in the disturbance rejection capability will be revealed. To satisfy the power flow control with variable switching frequency, the gain scheduling technique will be given. Finally, the analytical solution of overall system is revealed and verified by comparing with time-domain trace in circuit simulation model implemented in PLECS™ under different operating points. Furthermore, the proposed control design will be demonstrated by a scaled-down laboratory test bench.

## 2. Mode of Operation of Series Resonant Converter

The mode of operation of series resonant converter (SRC#) in Figure 2 is decided by the ratio between natural frequency

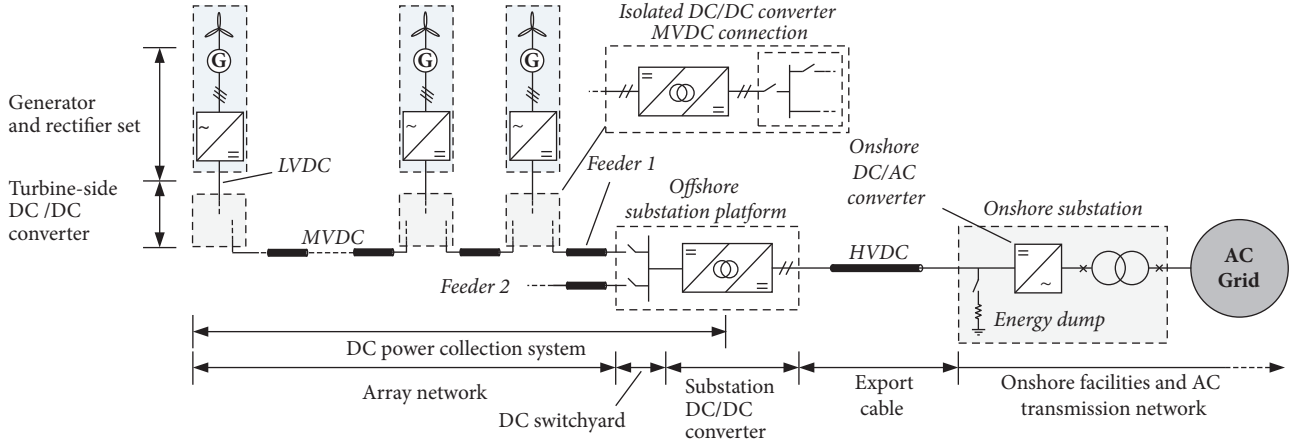


FIGURE 1: Generic configuration of the wind power plant with MVDC power collection.

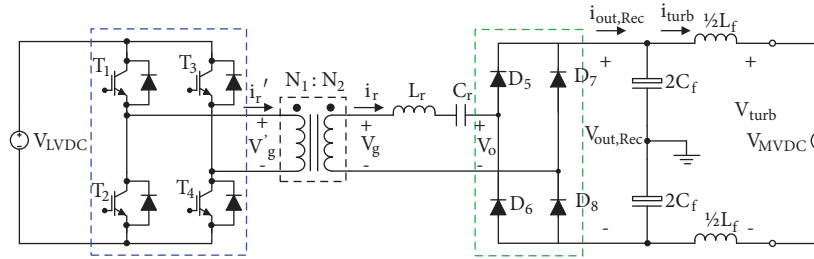


FIGURE 2: Circuit topology of series resonant converter (SRC#).

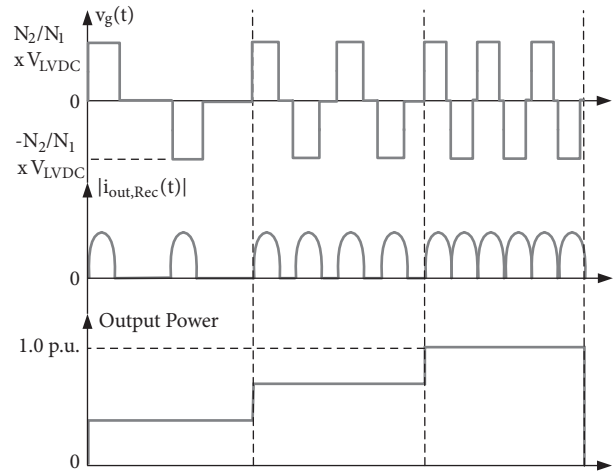


FIGURE 3: Frequency-depended power flow control of SRC#.

of tank ( $L_r$  and  $C_r$ ) and the switching frequency of H-bridge inverter: subresonant, resonant, and super resonant mode. In subresonant mode, the switching frequency of H-bridge inverter is lower than the natural frequency of tank. The resonant operating mode is selected when the switching frequency is equal to the natural frequency of tank. If converter's switching frequency is higher than the natural frequency of tank, the converter is operated in the super resonant mode [9].

Contrasting with the constant frequency with phase shift control which is normally applied for operation in super resonant mode, to achieve ZVS at turn-on, Figure 3 illustrates the concept of frequency-depended power flow control of SRC#. The converter leg of SRC# consisting of switches  $T_1$  and  $T_2$  is referred to as the leading leg and the one consisting of switches  $T_3$  and  $T_4$  is referred to as the lagging leg as indicated in Figure 2. Both converter legs operate at a 50% duty cycle [6, 9].

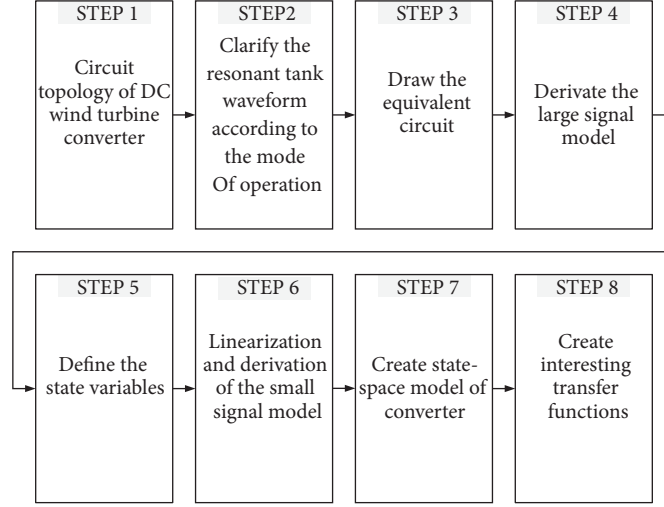


FIGURE 4: Flow chart of derivation of plant model of SRC#.

To achieve ZCS character at turn-off or minimize the turn-off current, the IGBT-based SRC# is designed to operate at subresonant continuous-conduction mode (subresonant CCM). This control design can drive the implemented phase shift having the same length as the resonant pulse without sacrificing the advantage of linear relation to the number of resonant pulses, as depicted in Figure 3. Compared to a traditional SRC with frequency control design in subresonant mode, therefore, the medium frequency transformer in the SRC# addressed in this paper can be designed for a higher frequency and avoids saturation for lower frequencies.

### 3. Discrete Time Domain Modelling Approach for Series Resonant Converter

Considering the efficiency, subresonant mode is selected for the mode of operation of SRC# for the DC wind turbine [5, 6]. Based on the circuit topology shown in Figure 2, Figure 5 illustrates the steady-state voltage and current waveforms of SRC# in subresonant mode, where  $\omega_s$  is the switching frequency ( $\omega_s = 2\pi \cdot f_s$ ) of SRC#. To apply the linear control theory to the SRC# control design, deriving the plant model of SRC# with the discrete time domain modelling approach includes the derivation of large-signal equations based on the interesting interval shown in Figure 5, linearization of discrete state equations, and derivation of small-signal transfer function. In the derivation, the voltages  $v_{MVDc}(t)$  and  $v_{LVDC}(t)$  are assumed to be discrete in nature, having the constant values  $V_{o(k)}$  and  $V_{g(k)}$  in interval of  $k_{th}$  event, and then switch to next states  $V_{o(k+1)}$  and  $V_{g(k+1)}$  at the start of  $(k+1)_{th}$  event. This procedure is only valid when the variation in  $v_o(t)$  or  $v_g(t)$  in the event is relatively smaller than its initial and final values [10].

With the discrete time domain modelling approach, (1) gives a linearized state-space model of SRC# in subresonant mode and the transfer functions between input state variables and the defined interesting states are shown in (3) and (5). To simplify the derivation, the output filter of SRC# (i.e.,  $L_f$

and  $C_f$ ) is neglected and only the DC component of output current diode rectifier  $i_{out,Rec}$  is selected as an output variable  $I_o$ .

To obtain the harmonic model of DC turbine converter, Figure 4 gives a complete flow chart of mathematical derivation of SRC# plant model, which describes how the SRC# plant model is obtained. First of all, the circuit topology and mode of operation are decided as shown in Figures 2 and 5 and then the equivalent circuit based on the switching sequence of transistors is generated in Figure 6. Based on the circuit topology shown in Figure 2, Figure 5 illustrates the voltage and current waveforms of SRC# n subresonant mode and the equivalent circuit for each event (switching interval) is given in Figure 6. According to Figure 6, the large signal model of converter is created (step 4) and then the interesting state variables (step 5) are defined to generate the small-signal equation and the space model of converter as in steps 6 and 7, respectively ([A], [B], [C], and [D]). Eventually, the converter plant model (power stage of converter) is established based on the interesting transfer function ( $g_1$ ,  $g_2$ , and  $g_3$ , in step 8). The correction of model (plant model) has been confirmed and the details of derivation are given in the Appendix.

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = [A] \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + [B] \begin{bmatrix} \tilde{\alpha} \\ \tilde{V}_g \\ \tilde{V}_o \end{bmatrix} \quad (1)$$

$$\tilde{I}_o = [C] \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + [D] \begin{bmatrix} \tilde{\alpha} \\ \tilde{V}_g \\ \tilde{V}_o \end{bmatrix}$$

where

$$\begin{aligned} \tilde{x}_1 &= \tilde{I}_r, \\ \tilde{x}_2 &= \tilde{v}_{cr} \end{aligned} \quad (2)$$

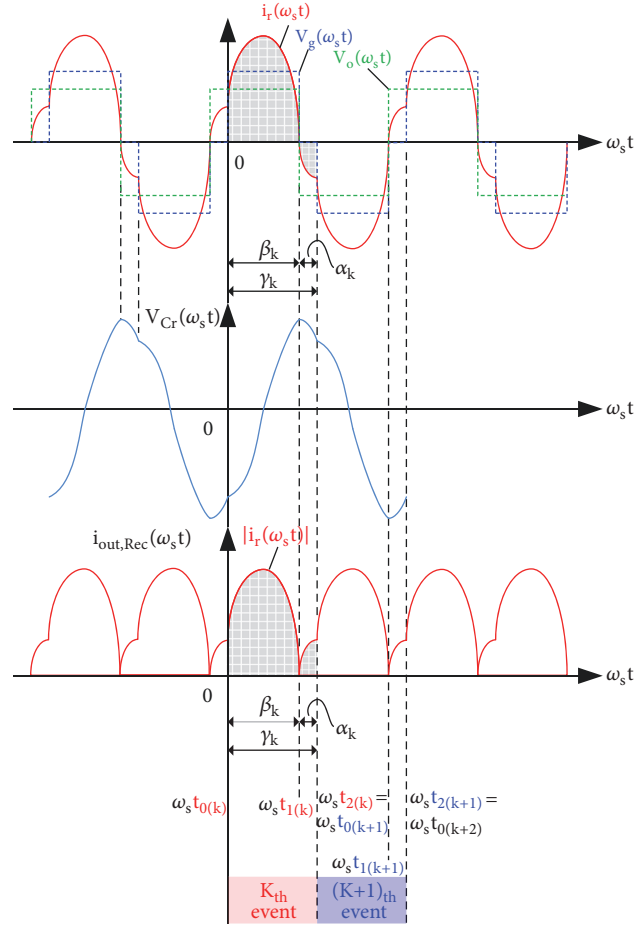


FIGURE 5: Resonant inductor current and resonant capacitor voltage waveforms of SRC# in subresonant CCM.

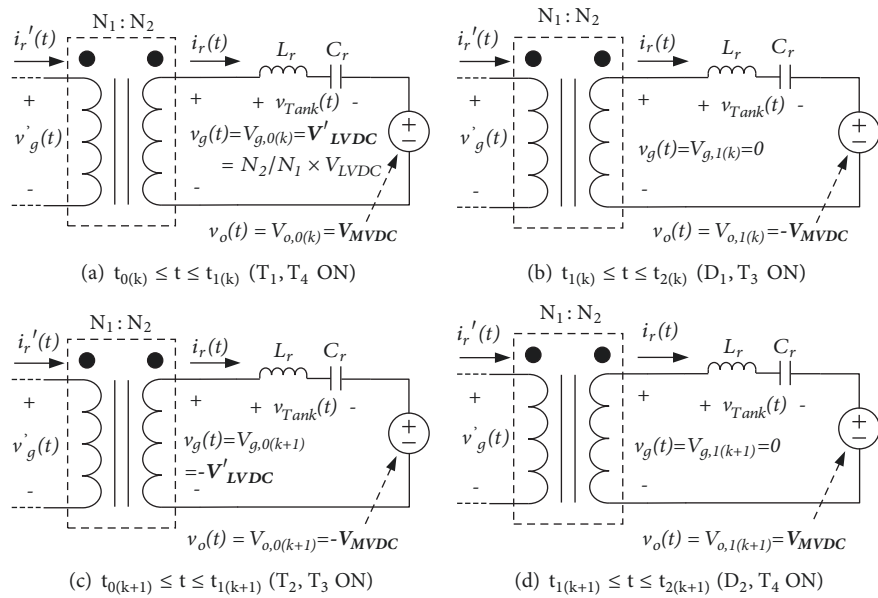


FIGURE 6: Equivalent circuit of SRC for large-signal analysis of conduction intervals in subresonant CCM.

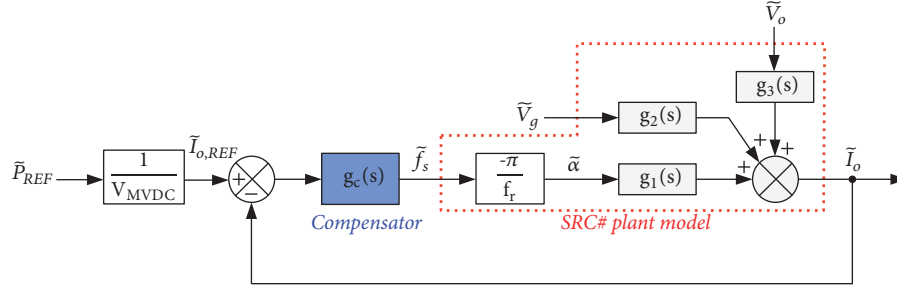


FIGURE 7: Small-signal control model of the series resonant converter SRC# in subresonant CCM.

and the derivation of  $[A]$ ,  $[B]$ ,  $[C]$ , and  $[D]$  matrixes is shown in the Appendix. Transfer functions between defined internal state variables and input states are given by

$$\tilde{X} = \begin{bmatrix} \tilde{I}_r \\ \tilde{v}_{cr} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} g_{xu,11} & g_{xu,12} & g_{xu,13} \\ g_{xu,21} & g_{xu,22} & g_{xu,23} \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{V}_g \\ \tilde{V}_o \end{bmatrix} \quad (3)$$

where

$$\begin{aligned} g_{xu,11} &= \left. \frac{\tilde{I}_r}{\tilde{\alpha}} \right|_{\tilde{V}_g(s)=0, \tilde{V}_o(s)=0}, \\ g_{xu,12} &= \left. \frac{\tilde{I}_r}{\tilde{V}_g} \right|_{\tilde{\alpha}(s)=0, \tilde{V}_o(s)=0}, \\ g_{xu,13} &= \left. \frac{\tilde{I}_r}{\tilde{V}_o} \right|_{\tilde{\alpha}(s)=0, \tilde{V}_g(s)=0}, \\ g_{xu,21} &= \left. \frac{\tilde{v}_{cr}}{\tilde{\alpha}} \right|_{\tilde{V}_g(s)=0, \tilde{V}_o(s)=0}, \\ g_{xu,22} &= \left. \frac{\tilde{v}_{cr}}{\tilde{V}_g} \right|_{\tilde{\alpha}(s)=0, \tilde{V}_o(s)=0}, \\ g_{xu,23} &= \left. \frac{\tilde{v}_{cr}}{\tilde{V}_o} \right|_{\tilde{\alpha}(s)=0, \tilde{V}_g(s)=0} \end{aligned} \quad (4)$$

and transfer functions between converter output current and input state variables are

$$\tilde{I}_o(s) = [g_1(s) \ g_2(s) \ g_3(s)] \begin{bmatrix} \tilde{\alpha} \\ \tilde{V}_g \\ \tilde{V}_o \end{bmatrix} \quad (5)$$

where the transfer functions  $g_1(s)$ ,  $g_2(s)$ , and  $g_3(s)$  can be obtained via

$$[g_1(s) \ g_2(s) \ g_3(s)] = C(SI - A)^{-1}B + D, \quad (6)$$

$$g_1(s) = \left. \frac{\tilde{I}_o(s)}{\tilde{\alpha}(s)} \right|_{\tilde{V}_g(s)=0, \tilde{V}_o(s)=0},$$

$$\begin{aligned} g_2(s) &= \left. \frac{\tilde{I}_o(s)}{\tilde{V}_g(s)} \right|_{\tilde{\alpha}(s)=0, \tilde{V}_o(s)=0}, \\ g_3(s) &= \left. \frac{\tilde{I}_o(s)}{\tilde{V}_o(s)} \right|_{\tilde{\alpha}(s)=0, \tilde{V}_g(s)=0} \end{aligned} \quad (7)$$

The transfer function  $g_1(s)$  describes how the output current  $\tilde{I}_o$  is influenced by the control input variable  $\tilde{\alpha}$  and the transfer functions  $g_2(s)$  and  $g_3(s)$  describe how the output current  $\tilde{I}_o$  is affected if any disturbance occurs in input voltage  $V_g$  ( $\propto V_{LVDC}$ ) and the output voltage  $V_o$  ( $\propto V_{MVDC}$ ). For example, the array network (MVDC grid) contains voltage harmonics. The transfer function  $g_3(s)$  can be used to evaluate the effect of voltage harmonics on the converter output current. Detailed derivation of the above linearized state-space model and the expression of elements in  $[A]$ ,  $[B]$ ,  $[C]$ , and  $[D]$  matrix in (1) have been revealed in the Appendix.

#### 4. Model-Based Closed-Loop Control Design

Figure 7 gives an overview of small-signal control model of SRC# based on the average plant model in (5) and (6). The control design of SRC# includes derivation of small-signal plant model and the design of the compensator  $g_c$ . The small-signal transfer functions of SRC# between converter output current and input state variables are given by (5) and (6), where the output current variation is the expression of linear combination of the three independence inputs. The relationship between  $\alpha$  and  $f_s$  in subresonant CCM in large-signal model is

$$\alpha = \left( \frac{1}{2f_s} - \frac{1}{2f_r} \right) 2\pi f_s = \pi - \frac{\pi}{f_r} f_s \quad (8)$$

where

$$f_r = \frac{1}{2\pi\sqrt{L_r C_r}} \quad (9)$$

By substituting the perturbation terms of small-signal analysis into expression in (8), the small-signal expression of  $\alpha$  and  $f_s$  can be obtained

$$\bar{\alpha} + \tilde{\alpha} = \pi - \frac{\pi}{f_r} (\bar{f}_s + \tilde{f}_s) \quad (10)$$

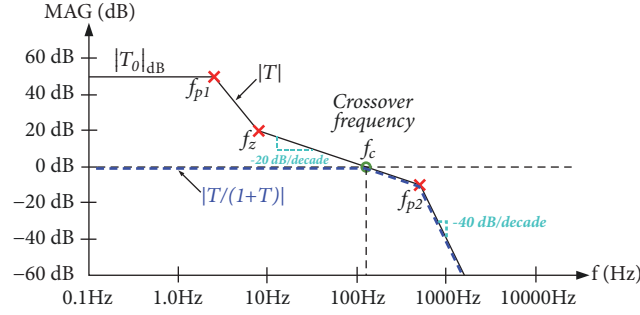


FIGURE 8: Illustration of magnitude asymptote of desired loop gain  $T(s)|_{\text{target}}$  (target curve of loop gain) [11].

where the AC component is

$$\tilde{\alpha} = -\frac{\pi}{f_r} \tilde{f}_s \quad (11)$$

Eventually, the system transfer function in Figure 5 can be expressed as

$$\begin{aligned} \tilde{I}_o(s) = & \frac{(1/V_{MVDC}) g_c (-\pi/f_r) g_1 \tilde{P}_{REF}}{1 + g_c (-\pi/f_r) g_1} \tilde{P}_{REF} \\ & + \frac{g_2}{1 + g_c (-\pi/f_r) g_1} \tilde{V}_g \\ & + \frac{g_3}{1 + g_c (-\pi/f_r) g_1} \tilde{V}_o \end{aligned} \quad (12)$$

Equation (12) can be further expressed as the following:

$$\tilde{I}_o(s) = \frac{1}{V_{MVDC}} \frac{T}{1+T} \tilde{P}_{REF} + \frac{g_2}{1+T} \tilde{V}_g + \frac{g_3}{1+T} \tilde{V}_o \quad (13)$$

with a loop gain.

$$T(s) = g_c \frac{-\pi}{f_r} g_1 \quad (14)$$

where the *loop gain* is defined by the product of gains around forward and feedback paths [11].

## 5. Disturbance Rejection Capability

The closed-loop control design of SRC# is implemented via the compensator  $g_c$ , which is applied to shape the loop gain of the system (i.e.,  $T(s)$ ). Considering the transfer function of output current given in (13), the relationship between  $\tilde{I}_o$  and  $\tilde{V}_g$  is shaped by closed-loop control as

$$\left. \frac{\tilde{I}_o(s)}{\tilde{V}_g(s)} \right|_{\tilde{P}_{REF}=0, \tilde{V}_o=0} = \frac{g_2}{1+T} \quad (15)$$

The variation in output current  $I_o$  caused by  $\tilde{V}_g$  can be alleviated by increasing the magnitude of the loop gain  $T(s)$  when the closed-loop control design is integrated with the SRC# plant model. The system transfer functions in (13) also

show that the variation reduction of  $I_o$  due to variation in MVDC network will benefit from a high loop gain  $T(s)$ :

$$\left. \frac{\tilde{I}_o(s)}{\tilde{V}_g(s)} \right|_{\tilde{P}_{REF}=0, \tilde{V}_o=0} = \frac{g_2}{1+T} \quad (16)$$

Furthermore, consider the tracking performance of output current control in (17).

$$\left. \frac{\tilde{I}_o(s)}{\tilde{P}_{REF}} \right|_{\tilde{V}_g=0, \tilde{V}_o=0} = \frac{1}{V_{MVDC}} \frac{T}{1+T} \quad (17)$$

Assume that a constant power reference  $P_{REF}$  is applied to the control loop with a constant MVDC source and a constant LVDC source. A large loop gain  $|T(s)|$  (i.e.,  $|T(s)| \gg 1$ ) can also make sure of a good DC current tracking performance as shown in (18)

$$\left. \frac{\tilde{I}_o}{\tilde{P}_{REF}} \right|_{\tilde{V}_g=0, \tilde{V}_o=0} \approx \frac{1}{V_{MVDC}} \quad (18)$$

Therefore, the objective of the compensator  $g_c$  is to govern the system with a desired loop gain (i.e.,  $T(s) = T(s)|_{\text{target}}$ ), where the deviation of desired loop transfer function  $T(s)|_{\text{target}}$  can be found by simply evaluating the magnitude asymptote in Figure 8:

$$\begin{aligned} T(s) &= g_c \frac{-\pi}{f_r} g_1 \\ &= T_0 \times \frac{(1 + s/\omega_z)}{(1 + s/Q\omega_{p1} + (s/Q\omega_{p1})^2)(1 + s/\omega_{p2})} \end{aligned} \quad (19)$$

Considering the desired loop gain  $T(s)|_{\text{target}}$  illustrated in Figure 8, the disturbance rejection capability of the output current for a frequency range below the crossover frequency ( $f_c$ ) can be improved with closed-loop control. For example, at the low frequency range ( $f < f_c$ ), the output current  $I_o$  is almost in direct proportion to the power reference signal  $P_{REF}$ .

$$\frac{T}{1+T} \approx \begin{cases} 1 & \text{for } f < f_c \quad (|T| \gg 1) \\ T & \text{for } f > f_c \quad (|T| \ll 1) \end{cases} \Rightarrow \quad (20)$$

$$\left. \frac{\tilde{I}_o(s)}{\tilde{P}_{REF}} \right|_{\tilde{V}_g=0, \tilde{V}_o=0} \approx \frac{1}{V_{MVDC}}$$



Furthermore, a high loop gain provides a good disturbance reduction to the variation on input voltage  $V_g$  and output voltage  $V_o$  by the factor  $1/|T|$ .

$$\frac{1}{1+T} \approx \begin{cases} \frac{1}{T} & \text{for } f < f_c \quad (|T| \gg 1) \\ 1 & \text{for } f > f_c \quad (|T| \ll 1) \end{cases} \Rightarrow$$

$$\left. \frac{\tilde{I}_o(s)}{\tilde{V}_g(s)} \right|_{\tilde{P}_{REF}=0, \tilde{V}_0=0} \approx \frac{g_2}{T}, \quad (21)$$

$$\left. \frac{\tilde{I}_o(s)}{\tilde{V}_0(s)} \right|_{\tilde{P}_{REF}=0, \tilde{V}_g=0} \approx \frac{g_3}{T}$$

Typically, the *crossover frequency*  $f_c$  should be less than approximately 10% of switching frequency of SRC# ( $f_c < 0.1f_s$ ) to limit the harmonics caused by PWM switching [11]. Based on (19), therefore, compensator  $g_c|_{OP}$  under a certain operating point (OP) can be expressed by

$$g_c|_{OP} = \frac{T|_{OP,target}}{(-\pi/f_r) g_1|_{OP}} = \frac{f_r}{-\pi}$$

$$\cdot \left. \frac{1}{g_1|_{OP}} \frac{T_0 \times (1+s/\omega_z)}{(1+s/Q\omega_{p1} + (s/Q\omega_{p1})^2)(1+s/\omega_{p2})} \right|_{OP,target} \quad (22)$$

Equations (23)–(27) summarize the parameters (i.e.,  $Q$ ,  $\omega_{p1}$ ,  $\omega_{p2}$ ,  $\omega_z$ ,  $\omega_c$ , and  $\theta$ ) which are used to shape the loop gain  $T(s)$  via the compensator  $g_c$ . The crossover frequency  $f_c$  and the low-frequency pole at  $f_{p1}$  are defined as

$$f_c = 0.1f_s, \quad (23)$$

$$f_{p1} = \frac{1}{4.5} f_c \quad (24)$$

The low-frequency zero at  $f_z$  and high-frequency pole at  $f_{p2}$  can be chosen according to crossover frequency  $f_c$  and required phase margin  $\theta$  as follows:

$$f_z = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}, \quad (25)$$

$$f_{p2} = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}} \quad (26)$$

where the angle  $\theta$  is a phase lead angle of compensator at  $f_c$ . The DC gain of target loop gain  $T(s)|_{target}$  is

$$T_o = \left( \frac{f_c}{f_{p1}} \right)^2 \times \sqrt{\frac{f_z}{f_{p2}}} \quad (27)$$

The *Q-factor* is used to characterize the transient response of closed-loop system. Using a high *Q-factor* can increase the dynamic response during transient, but it can also cause overshoot and ringing on power devices. In practical application, the *Q-factor* must be sufficiently low to keep enough

TABLE 1: Parameters of SRC# plant model.

Low voltage DC ( $V_{LVDC}$ )	4.04 (kV <sub>DC</sub> )
Medium voltage DC ( $V_{MVDC}$ )	100.0 (kV <sub>DC</sub> )
Transformer winding voltage ratio ( $N_1:N_2$ )	1: 25
Rated output power $P_{out}$	10 (MW)
Resonant inductor $L_r$	78.1 (mH)
Resonant capacitor $C_r$	0.25 (uF)

phase margins and alleviate voltage and current stress on power devices [11]. Additionally, since the power flow control of SRC# depends on the control of switching frequency  $f_s$ , the parameters of target curve and the coefficient of transfer function  $g_c$  have to be changed according to different operating points (different output powers). To make sure that the compensator  $g_c$  can match with different output power requirements, therefore, a gain scheduling approach is proposed which will be revealed in the next section.

## 6. Design of Digital Gain Scheduling Controller

Gain scheduling controller is designed to access the parameter of compensator  $g_c$  in real time and then adjust it based on the different operating points. Figure 9 gives a complete digital controller of SRC# based on the small-signal control model and the bilinear transformation. The digital controller of SRC# consists of a small-signal controller, a gain scheduling controller, a feedforward control loop, and a DC component calculator ( $I_o$  calculator). The controller is implemented in z-domain with a variable interrupt frequency  $f_{int}$  ( $f_{int} \propto$  switching frequency  $f_s$ ). With the bilinear transform, the general form of the discrete-time representation of the compensator  $g_c$  can be expressed as

$$g_c(z) = \frac{a_5 z^5 + a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0}{b_5 z^5 + b_4 z^4 + b_3 z^3 + b_2 z^2 + b_1 z + b_0} \quad (28)$$

where coefficients  $a_n$  and  $b_n$  ( $n=0\sim5$ ) are used to specify the coefficients of numerator and denominator.

To design the gain scheduling controller, coefficients  $a_n$  and  $b_n$  in (28) are evaluated under different operating points (i.e., different output power) with (22)–(27). A trend in the variation of each coefficient (i.e.,  $P_{REF}$  vs.  $a_n$  and  $b_n$ ) is recorded and then is formulated via the polynomial approximation as shown in Figures 12 and 13 which will be discussed in the next section (Section 7). Eventually, the coefficient of  $g_c(z)$  for SRC# in the subresonant CCM can be adjusted by a continuous function such as  $a_n = f(P_{REF})$  and  $b_n = f(P_{REF})$  in real time to avoid any potential turbulences caused by gain-changing.

## 7. Verification of Closed-Loop Control Design

With the SRC# topology in Figure 2 and the controller shown in Figure 9, Tables 1 and 2 give the parameters used in the state-space model and circuit simulation models (tools)



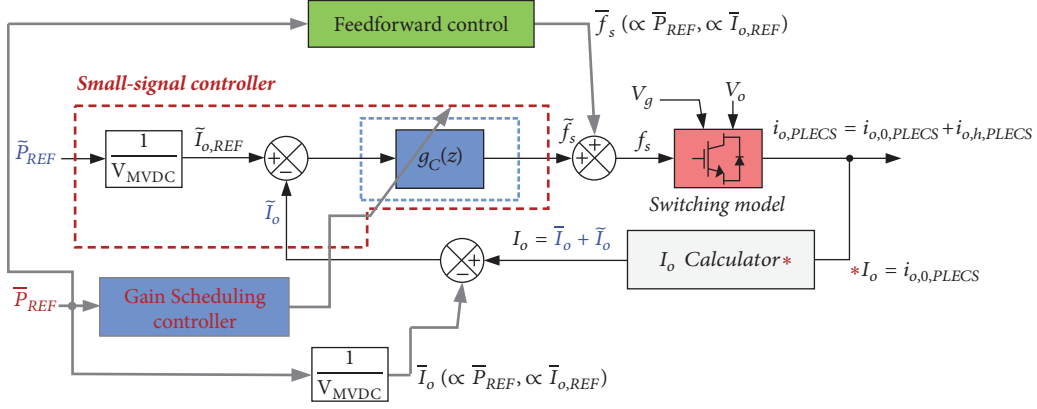


FIGURE 9: Control block of the series resonant converter SRC# in z-domain.

TABLE 2: Specifications of digital controller.

Switching frequency $f_s$	1.0k (Hz) (full load)
Interrupt frequency of digital controller $f_{int}$ (i.e. $f_{int} = 2f_s$ )	2.0k (Hz) (full load)
Q-factor	1.0
Phase margin $\theta$	52°
Power reference signal $P_{REF}$	10MW (full load)
Sampling rate of A/D conversion	1M (Hz)
Duty cycle	50%

for verifying the validation of overall system in z-domain. The control model in the subresonant CCM is verified to identify the accuracy of proposed small-signal model, and then the results of coefficient assessment of  $g_c(z)$  with the gain scheduling controller are integrated with control loop and are tested by a ramp-power reference.

By applying a +0.5% stepping perturbation to all input state variables, Figures 10 and 11 give the analytical solutions of small-signal model of SRC# and the results obtained from the time-domain switching model implemented in PLECS™. The SRC# with closed-loop control is commanded to deliver around 9.0MW DC power and 7.5MW DC power to MVDC network, respectively. Figures 10 and 11 show that both the steady state and transient state in the analytical model match with the results generated by switching model. Therefore, dynamics of SRC# switching model can be predictable and controlled with the proposed small-signal model.

Figures 12 and 13 give the result of coefficient assessment of  $g_c(z)$  for the design of the gain scheduling controller. Based on (28), the trend in the variation of coefficients  $a_n$  and  $b_n$  in subresonant CCM from 5.75MW to 10MW (0.5MW/step) is identified and then the variation of each coefficient is approximated with a 3<sup>rd</sup> polynomial (i.e.,  $a_n(P_{REF})|_{PolyFit}$  and  $b_n(P_{REF})|_{PolyFit}$ ). According to the variation in output power reference  $P_{REF}$ , the gain scheduling controller accesses the polynomial  $g_c(z)$  to regulate its coefficient in real time. To

TABLE 3: Specifications of laboratory test bench.

Low voltage DC source ( $V_{LVDC}$ )	216 ( $V_{DC}$ )
DC component of medium voltage source ( $V_{MVDC,0}$ )	400 ( $V_{DC}$ )
Transformer winding voltage ratio ( $N_1:N_2$ )	1: 2
Rated output power $P_{out}$	550 (W)
Resonant inductor $L_r$	20.0 (mH)
Resonant capacitor $C_r$	1.0 (uF)
Switching frequency $f_s$	800 (Hz)
Output filter inductor $L_f$	2.5 (mH)
Output filter capacitor $C_f$	1.0 (mF)
Resistive load $R_{load}$	125 ( $\Omega$ )
Interrupt frequency $f_{int}$ (i.e. $f_{int} = 2f_s$ )	1.6k (Hz)
Sampling rate of A/D conversion	1M (Hz)
Duty cycle	50%

evaluate the adequacy of control design of overall system, finally, the time-trace simulation of output power flow control is given in Figure 14 with a ramp-power reference  $P_{REF}$  from 0.1MW to 10MW, and vice versa. The results show that the output current/power ( $I_o$ ) of the series resonant converter can be well controlled when magnitude output powers references are changed.

## 8. Laboratory Test Results

To verify the control design, first the circuit simulation is carried out with circuit simulation tool of PLECS™, and then the controller is implemented in a scaled-down laboratory test bench. The circuit configuration of test bench and the corresponding parameters are shown in Figure 15 and Table 3, respectively, where the MVDC network is simulated

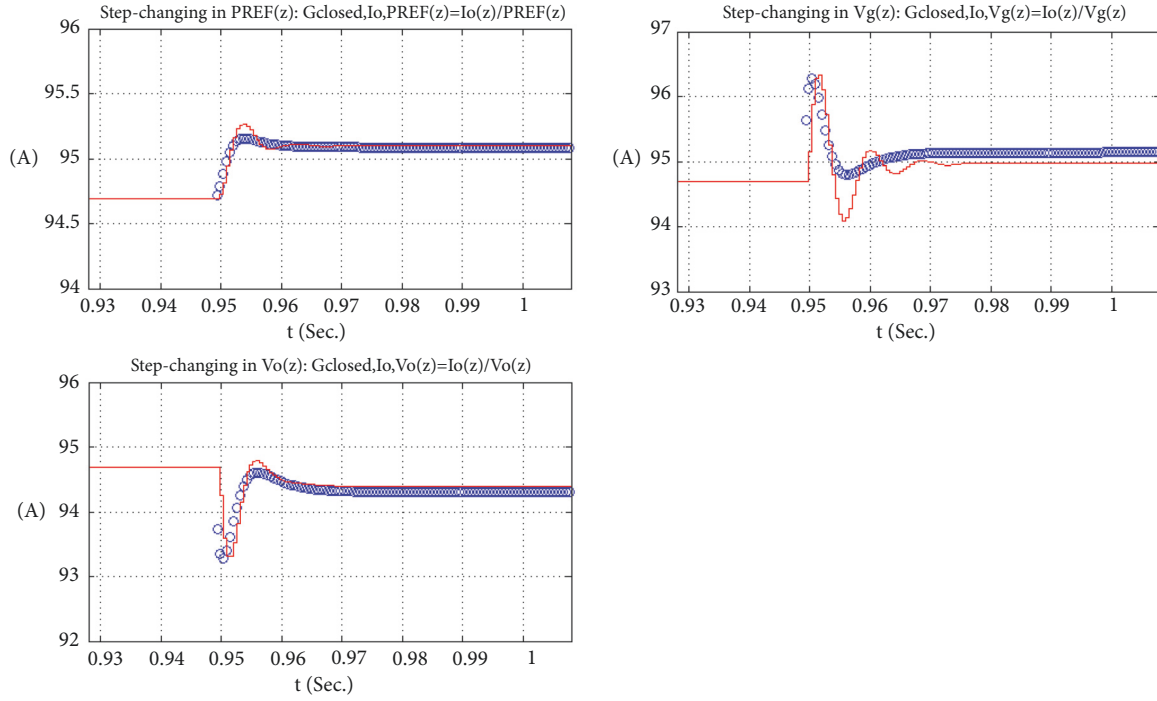


FIGURE 10: Dynamics of output current  $I_o$  generated by both the switching model and derived state-space model with the closed-loop controller when +0.5% of step-changing is applied in  $P_{REF}$ ,  $V_g$ , and  $V_o$ , respectively ( $P_{REF}$ : 9.0MW  $\rightarrow$  9.045MW,  $V_g$ : 101.01kV<sub>DC</sub>  $\rightarrow$  101.515kV<sub>DC</sub>,  $V_o$ : 100.0kV<sub>DC</sub>  $\rightarrow$  100.5kV<sub>DC</sub>; blue circle: dynamic of state-space model in z-domain, red line: dynamic of electrical signal in PLECS circuit model, the interrupt time of digital controller:  $T_{int} = 1/(2 \times f_{s,lop}) = 1/(2 \times 900\text{Hz})$  sec).

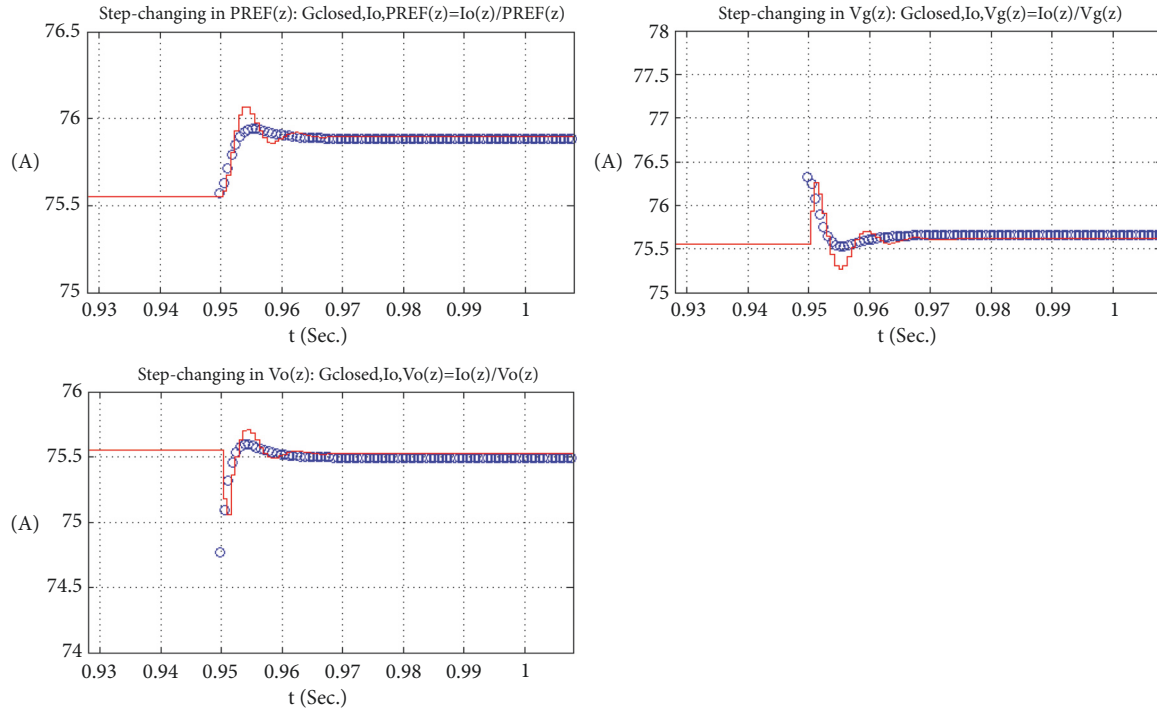


FIGURE 11: Dynamics of output current  $I_o$  generated by both the switching model and derived state-space model with the closed-loop controller when +0.5% of step-changing is applied in  $P_{REF}$ ,  $V_g$ , and  $V_o$ , respectively ( $P_{REF}$ : 7.5MW  $\rightarrow$  7.5375MW,  $V_g$ : 101.01kV<sub>DC</sub>  $\rightarrow$  101.515kV<sub>DC</sub>,  $V_o$ : 100.0kV<sub>DC</sub>  $\rightarrow$  100.5kV<sub>DC</sub>; blue circle: dynamic of state-space model in z-domain, red line: dynamic of electrical signal in PLECS circuit model, the interrupt time of digital controller:  $T_{int} = 1/(2 \times f_{s,lop}) = 1/(2 \times 750\text{Hz})$  sec).

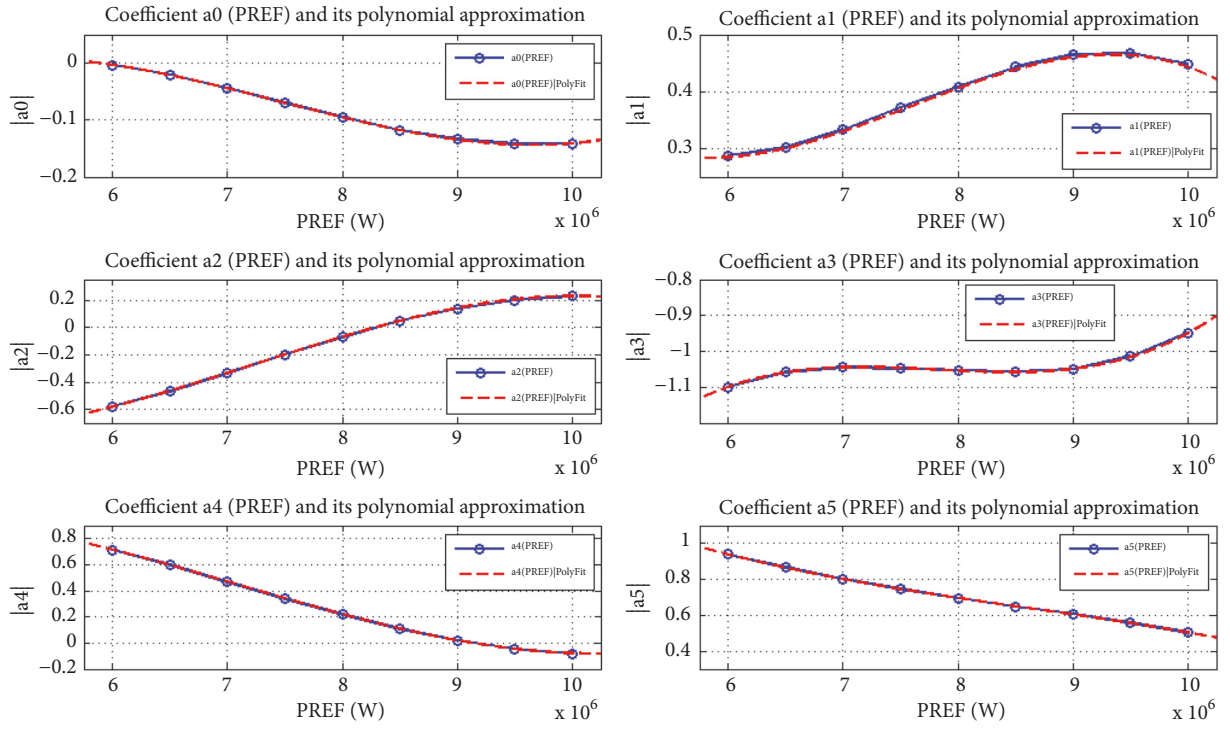


FIGURE 12: Design of gain scheduling controller: piecewise continuous functions of numerator of  $g_c(z)$  and its polynomial approximation ( $3^{\text{rd}}$ ) in subresonant CCM.

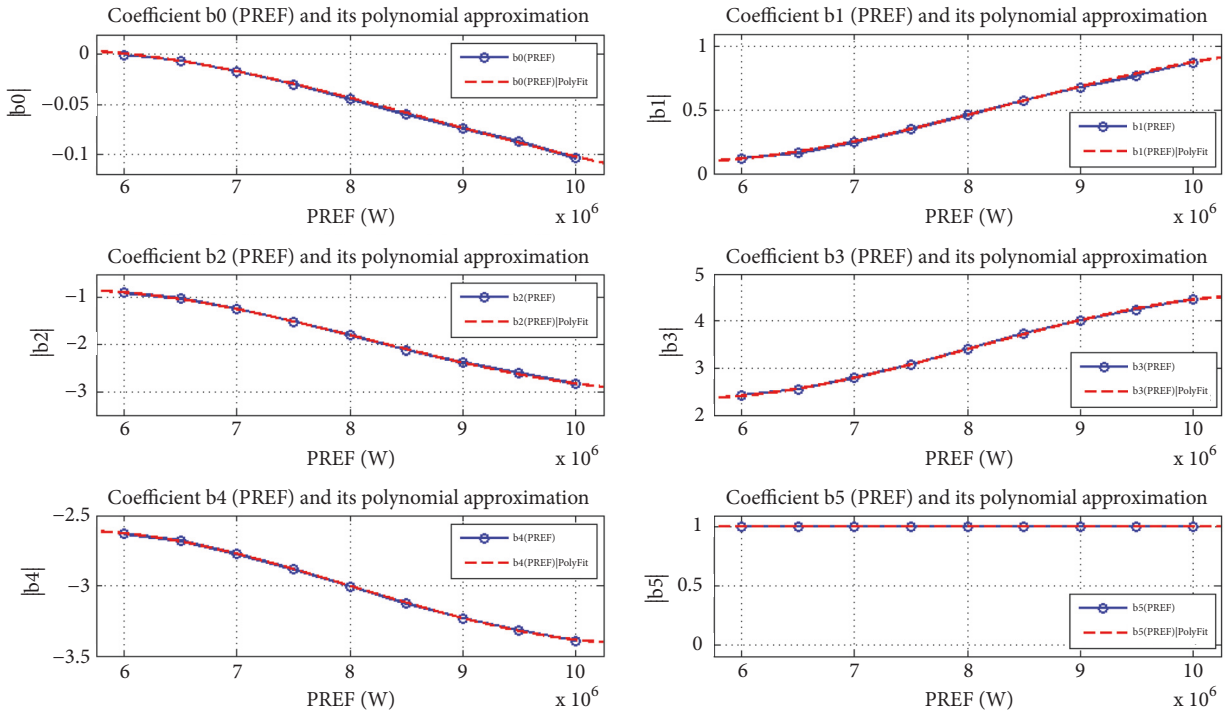


FIGURE 13: Design of gain scheduling controller: piecewise continuous functions of denominator of  $g_c(z)$  and its polynomial approximation ( $3^{\text{rd}}$ ) in subresonant CCM.

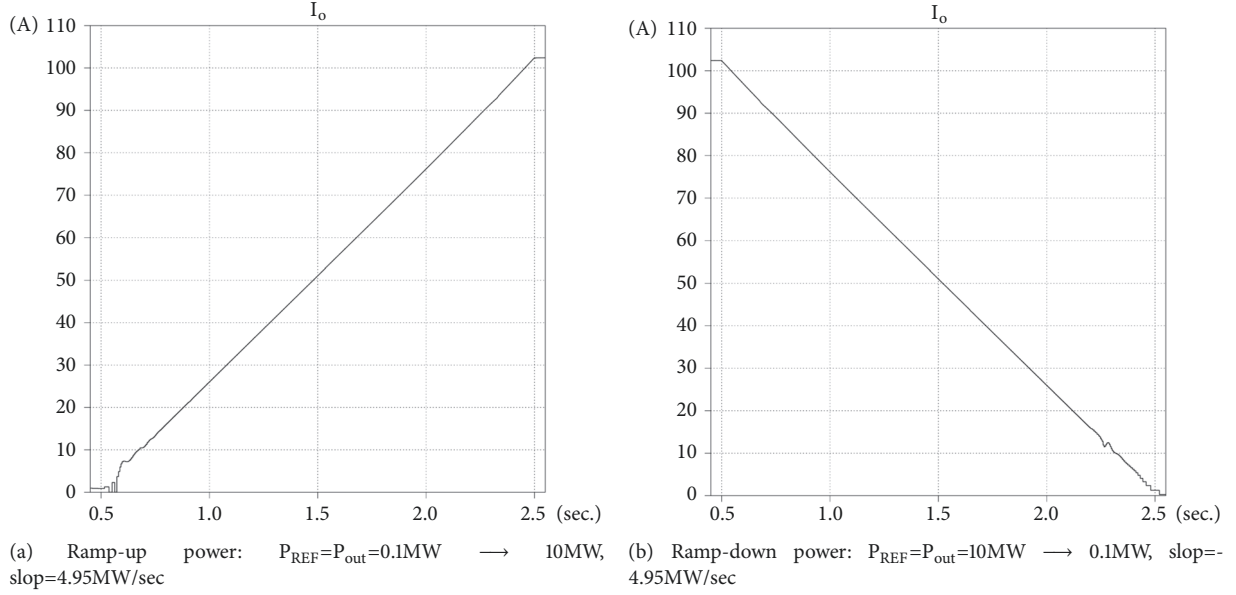


FIGURE 14: Output current ( $I_o$ ) of the series resonant converter with a ramp-power reference  $P_{REF}$  to verify the design of gain scheduling controller and demonstrate the start-up process of DC wind converter.

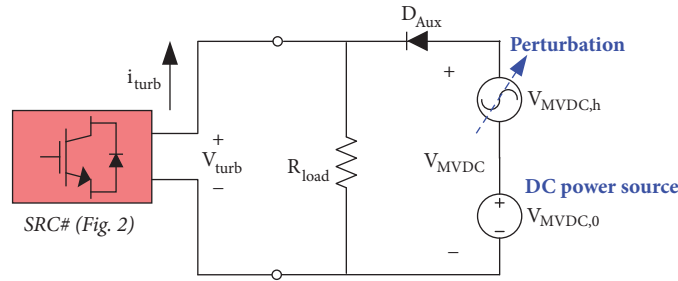


FIGURE 15: Configuration of laboratory test bench ( $V_{MVDC,0}$ : DC component of medium voltage power source,  $V_{MVDC,h}$ : perturbation source).

by a unidirectional power flow DC power source with a controllable perturbation.

Figures 16(a) and 16(b) depict the system response when a positive and a negative step perturbation (0.01p.u) in MVDC network are applied, respectively. The control design exhibits a close behavior in either simulation or experimental test. There is some small tracking error during the transient between the simulation and test results. This usually is caused by the estimated error of components and stray inductance which is not considered in simulation model. Figure 16(c) represents how the output current behaves when a step-change (0.26p.u) is applied in the power reference signal  $P_{REF}$ . Under the proposed control law for SRC#, both the simulation and test result show that the DC component of DC turbine output current ( $\tilde{i}_{turb}$ ) tracking performance can be guaranteed. However, a small oscillation ( $\approx 40\text{Hz}$ ) during the transient of step-change of power reference signal in the experimental test is observed due to the series diode  $D_{Aux}$  (in Figure 15) which is reverse-biased at this test occasion.

## 9. Conclusion

A model-based control design of SRC# for DC wind power plant based on small-signal plant model in the discrete time-domain modelling is revealed. This paper continues with the modelling of SRC# given in the Appendix and mainly addresses the closed-loop control design for the system. The control design process contains the derivation of state-space plant model, design of closed-loop control structure, and design of gain scheduling controller. Compared with the traditional frequency-dependend power flow control which relied on open-loop structure, the SRC# with the closed-loop structure can gain a better disturbance rejection capability for the output power control. The verification of proposed digital controller including plant model is addressed in both the analytical model and the time-domain circuit simulation implemented in PLECS™ in Section 7 by evaluating the SRC# with the stepping-perturbation under the subresonant CCM. Furthermore, gain scheduling approach is implemented by the polynomial approximation and tested under different

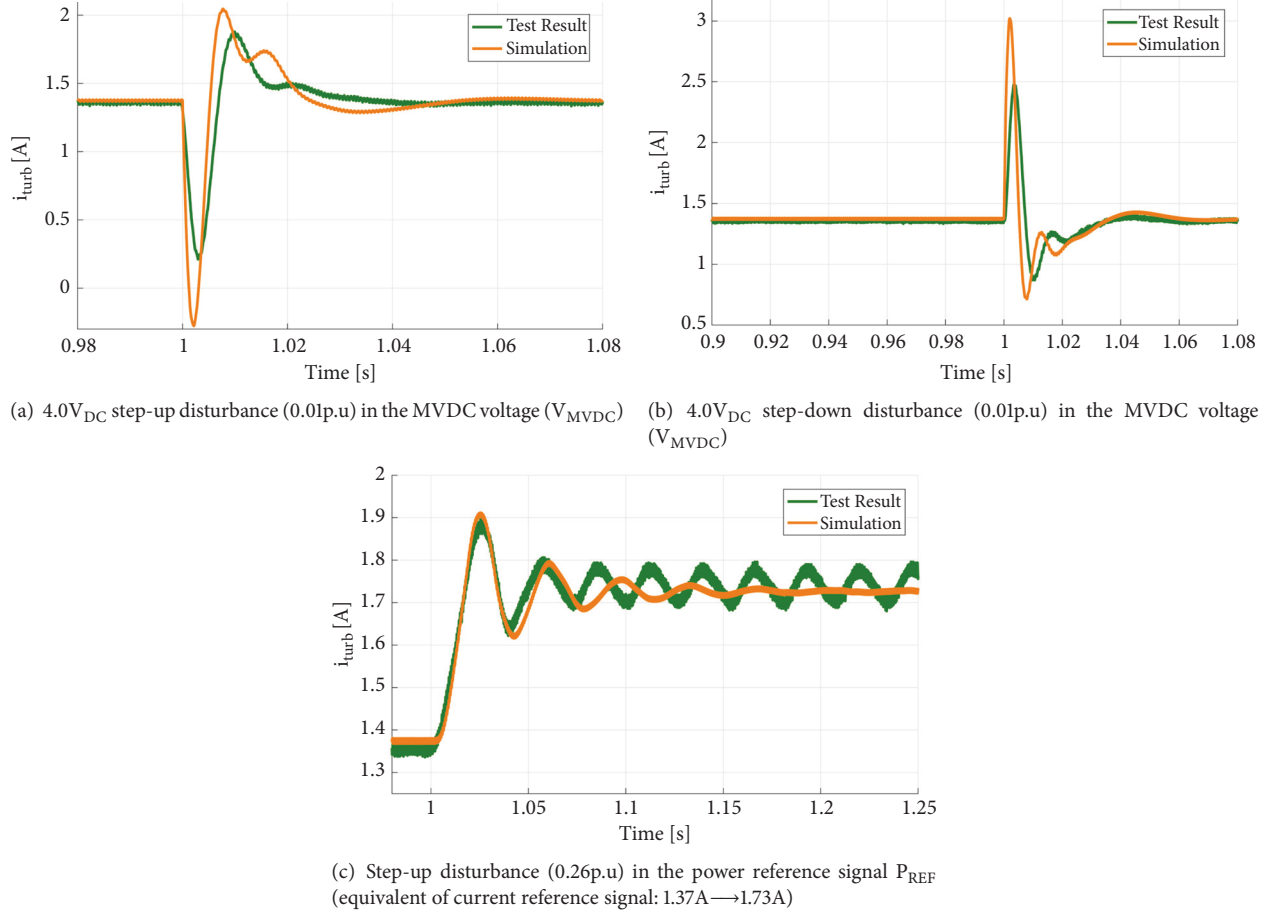


FIGURE 16: Dynamic response of output current of DC wind turbine converter ( $i_{turb}$ ) when a step-up/-down disturbance is injected in system at  $t=1.0$  [s].

operating points (different output powers). Integrating the gain scheduling controller with closed-loop structure enables the system to automatically adjust parameters of controller in real time to satisfy different output power requirements without sacrificing the control performance. Finally, Section 8 shows that all the test results give enough supporting arguments to the proposed control design.

## Appendix

The objective of the study is to understand the harmonics distribution of offshore DC wind farm and how the DC wind turbines are affected by harmonics from MVDC grid. This section summarizes the derivation of plant model of DC wind turbine based on the discrete time domain modelling approach (discrete time domain modelling approach [10], steps 1-8) which can help the reader to reach the plant model of DC wind turbine (SRC#) and then conduct control design of DC wind turbine. The following discussion will give a complete derivation process including the corresponding flow chart of the derivation of SRC# plant model given in Figure 4.

*Steps 1 -3: Decide the Circuit Topology of DC Turbine Converter, Resonant Tank Waveform, and Equivalent Circuit.* Steps

1-3 describe the circuit topology of SRC# (DC wind turbine converter) and mode of operation, which is operated in subresonant CCM as in Figures 2 and 5. The corresponding equivalent circuit for the SRC# in subresonant CCM is given in Figure 6, where the waveform is divided by different time zone (different switching sequence) based on the *discrete time domain modelling* approach proposed by King, R. J. [10]. Those figures (Figures 2, 5, and 6) are used to generate the large signal model of SRC#.

*Step 4: Large Signal Model.* Based on Figure 6, the objective of derivation of large-signal model is to express the final value of interesting state variables in each switching interval with the initial values. The procedure is only valid when the variation in output voltage  $v_o(t)$  (MVDC grid voltage) or input voltage  $v_g(t)$  (LVDC voltage) in the event (switching) is relatively smaller than its initial and final values [10]. Equations (A.1) to (A.16) give the derivation of large-signal model of resonant inductor current  $i_r(t)$  and resonant capacitor voltage  $v_{Cr}(t)$  and their end values at  $k_{th}$  event in terms of initial values of  $k_{th}$  event.

For  $t_{0(k)} \leq t \leq t_{1(k)}$  ( $T_1, T_4$  ON)

$$v_g = L_r \frac{di_r}{dt} + v_{Cr} + v_o,$$

$$i_r = C_r \frac{dv_{Cr}}{dt} \quad (\text{A.1})$$

where

$$\begin{aligned} v_g &= V_{g,0(k)}, \\ v_o &= V_{o,0(k)} \end{aligned} \quad (\text{A.2})$$

The resonant inductor current  $i_r(t)$  and resonant capacitor voltage  $v_{Cr}(t)$  can be obtained by solving (A.1).

$$i_r = \frac{1}{Z_r} (V_{g,0(k)} - V_{o,0(k)} - V_{Cr,0(k)}) \sin(\omega_r t) + I_{r,0(k)} \cos(\omega_r t), \quad (\text{A.3})$$

$$v_{Cr} = V_{g,0(k)} - (V_{g,0(k)} - V_{o,0(k)} - V_{Cr,0(k)}) \cos(\omega_r t) + I_{r,0(k)} Z_r \sin(\omega_r t) - V_{o,0(k)} \quad (\text{A.4})$$

where

$$\begin{aligned} Z_r &= \sqrt{\frac{L_r}{C_r}}, \\ \omega_r &= \frac{1}{\sqrt{L_r C_r}} \end{aligned} \quad (\text{A.5})$$

At time  $t = t_{1(k)}$  the tank current  $i_r$  makes a zero crossing, commutating  $T_1$  and  $T_4$  off and turning on  $D_1$  and  $T_3$ . Therefore,

$$\begin{aligned} I_{r,1(k)} &= i_r(t_{1(k)}) \\ &= \frac{1}{Z_r} (V_{g,0(k)} - V_{o,0(k)} - V_{Cr,0(k)}) \sin(\omega_r t_{1(k)}) + I_{r,0(k)} \cos(\omega_r t_{1(k)}) = 0 \end{aligned} \quad (\text{A.6})$$

where

$$\omega_r t_{1(k)} = \frac{\omega_r}{\omega_s} \beta_K = \omega_{rs} \cdot \beta_K \quad \text{for } t_{0(k)} = 0 \quad (\text{A.7})$$

$$\tan(\omega_{rs} \beta_K) = \frac{-I_{r,0(k)} Z_r}{(V_{g,0(k)} - V_{o,0(k)} - V_{Cr,0(k)})}, \quad (\text{A.8})$$

$$0 < (\omega_{rs} \beta_K) \leq \pi, \quad (\text{A.9})$$

$$t_{1(k)} = \frac{\beta_K}{\omega_s}$$

$$\begin{aligned} V_{Cr,1(k)} &= v_{Cr}(t_{1(k)}) \\ &= V_{g,0(k)} - (V_{g,0(k)} - V_{o,0(k)} - V_{Cr,0(k)}) \cdot \cos(\omega_{rs} \beta_K) + I_{r,0(k)} Z_r \cdot \sin(\omega_{rs} \beta_K) - V_{o,0(k)} \end{aligned} \quad (\text{A.10})$$

For  $t_{1(k)} \leq t \leq t_{2(k)}$  ( $D_1, T_3$  ON)

$$i_r(t') = \frac{-1}{Z_r} (V_{o,1(k)} + V_{Cr,1(k)}) \sin(\omega_r t') \quad (\text{A.11})$$

where

$$t' = t - t_{1(k)}, \quad (\text{A.12})$$

$$v_{Cr}(t') = (V_{o,1(k)} + V_{Cr,1(k)}) \cos(\omega_r t') - V_{o,1(k)} \quad (\text{A.13})$$

where

$$\begin{aligned} I_{r,1(k)} &= 0, \\ V_{g,1(k)} &= 0, \\ V_{o,1(k)} &= -V_{MVD C} = -V_{o,0(k)} \end{aligned} \quad (\text{A.14})$$

Eventually, the inductor current  $i_r(t)$  and capacitor voltage  $v_{Cr}(t)$  at time  $t=t_{2(k)}$  can be represented by

$$\begin{aligned} I_{r,2(k)} &= [-\sin(\omega_{rs} \beta_K) \cdot \sin(\omega_{rs} \alpha_K)] \cdot I_{r,0(k)} \\ &+ \left[ -\frac{1}{Z_r} \cdot \cos(\omega_{rs} \beta_K) \cdot \sin(\omega_{rs} \alpha_K) \right] \cdot V_{Cr,0(k)} \\ &+ \left[ \frac{2}{Z_r} \cdot \sin(\omega_{rs} \alpha_K) + \frac{-1}{Z_r} \cdot \cos(\omega_{rs} \beta_K) \cdot \sin(\omega_{rs} \alpha_K) \right] \cdot V_{o,0(k)} + \left[ \frac{-1}{Z_r} \cdot \sin(\omega_{rs} \alpha_K) + \frac{1}{Z_r} \cdot \cos(\omega_{rs} \beta_K) \cdot \sin(\omega_{rs} \alpha_K) \right] \cdot V_{g,0(k)}, \end{aligned} \quad (\text{A.15})$$

$$\begin{aligned} V_{Cr,2(k)} &= [Z_r \cdot \sin(\omega_{rs} \beta_K) \cdot \cos(\omega_{rs} \alpha_K)] \cdot I_{r,0(k)} \\ &+ [\cos(\omega_{rs} \beta_K) \cdot \cos(\omega_{rs} \alpha_K)] \cdot V_{Cr,0(k)} + [-2 \cdot \cos(\omega_{rs} \alpha_K) + \cos(\omega_{rs} \beta_K) \cdot \cos(\omega_{rs} \alpha_K) + 1] \\ &\cdot V_{o,0(k)} + [\cos(\omega_{rs} \alpha_K) - \cos(\omega_{rs} \beta_K) \cdot \cos(\omega_{rs} \alpha_K)] \cdot V_{g,0(k)} \end{aligned} \quad (\text{A.16})$$

where

$$\omega_s \cdot (t_{2(k)} - t_{1(k)}) = \alpha_K \quad (\text{A.17})$$

The large-signal expression of resonant inductor current  $i_r(t)$  and resonant capacitor voltage  $v_{Cr}(t)$  in  $(k+1)_{th}$  event ( $t_{0(k+1)} \leq t \leq t_{1(k+1)}$  and  $t_{1(k+1)} \leq t \leq t_{2(k+1)}$ ) can be obtained with the same process as derivation of equations, as in (A.1)-(A.16).

*Steady-State Solution of Large-Signal Model.* Equation (A.18) gives the conditions for calculating steady-state solution (operating points) of discrete state equation.

$$\begin{aligned} I_{r,2(k)} &= -I_{r,0(k)}, \\ V_{Cr,2(k)} &= -V_{Cr,0(k)} \end{aligned} \quad (\text{A.18})$$



By substituting (A.18) into (A.15) and (A.16), the steady-state solution of  $I_{r,0(k)}$  and  $V_{Cr,0(k)}$  can be expressed in terms of  $V_{o,0(k)}$ ,  $V_{g,0(k)}$ ,  $\beta_k$ , and  $\alpha_k$ :

$$\bar{I}_r = I_{r,0(k)} = f(V_{o,0(k)}, V_{g,0(k)}, \beta_k, \alpha_k), \quad (\text{A.19})$$

$$\bar{V}_{Cr} = V_{Cr,0(k)} = f(V_{o,0(k)}, V_{g,0(k)}, \beta_k, \alpha_k) \quad (\text{A.20})$$

where the overbar is used to indicated the steady-state value of interesting state variables.

To simplify the derivation, the output filter of SRC (i.e.,  $L_f$  and  $C_f$ ) is neglected due to very slow dynamics in voltage and current compared with the resonant inductor current and resonant capacitor and only the DC component of output current diode rectifier  $i_{out,Rec}$  is selected as an output variable  $i_o$ . Therefore, during the  $K_{th}$  event, the output current equation delivered by the SRC is expressed as

$$\begin{aligned} i_o &= \frac{1}{\gamma_k} \int_0^{\beta_k} i_{out,Rec}(\theta_s) d\theta_s + \frac{1}{\gamma_k} \int_{\beta_k}^{\gamma_k} i_{out,Rec}(\theta_s) d\theta_s \\ &= \frac{1}{\gamma_k} \cdot \left\{ \frac{1}{\omega_{rs}} \sin(\omega_{rs}\beta_k) + \frac{1}{\omega_{rs}} \sin(\omega_{rs}\beta_K) \right. \\ &\quad \cdot [1 - \cos(\omega_{rs}\alpha_k)] \Big\} \cdot I_{r,0(k)} + \frac{1}{\gamma_k} \\ &\quad \cdot \left\{ -\frac{1}{\omega_{rs}} \frac{1}{Z_r} [1 - \cos(\omega_{rs}\beta_k)] \right. \\ &\quad \left. + \frac{1}{\omega_{rs}} \frac{1}{Z_r} \cos(\omega_{rs}\beta_K) \cdot [1 - \cos(\omega_{rs}\alpha_k)] \right\} \\ &\quad \cdot V_{Cr,0(k)} + \frac{1}{\gamma_k} \cdot \left\{ -\frac{1}{\omega_{rs}} \frac{1}{Z_r} [1 - \cos(\omega_{rs}\beta_k)] \right. \\ &\quad \left. + \frac{1}{\omega_{rs}} \frac{1}{Z_r} (\cos(\omega_{rs}\beta_K) - 2) \cdot [1 - \cos(\omega_{rs}\alpha_k)] \right\} \\ &\quad \cdot V_{o,0(k)} + \frac{1}{\gamma_k} \cdot \left\{ \frac{1}{\omega_{rs}} \frac{1}{Z_r} [1 - \cos(\omega_{rs}\beta_k)] \right. \\ &\quad \left. + \frac{1}{\omega_{rs}} \frac{1}{Z_r} (1 - \cos(\omega_{rs}\beta_K)) \cdot [1 - \cos(\omega_{rs}\alpha_k)] \right\} \\ &\quad \cdot V_{g,0(k)} \end{aligned} \quad (\text{A.21})$$

where

$$\begin{aligned} \theta_s &= \omega_s t, \\ \alpha_k &= \gamma_k - \beta_k \end{aligned} \quad (\text{A.22})$$

and  $I_{r,0(k)}$  is the initial value of Inductor current,  $V_{Cr,0(k)}$  is the initial value of capacitor voltage,  $V_{o,0(k)}$  is the initial value of rectifier output voltage, and  $V_{g,0(k)}$  is then initial value of input voltage of SRC#.  $Z_r (= \sqrt{L_r/C_r})$  is characteristic impedance defined by parameter of resonant tanks,  $\alpha_k (= \gamma_k - \beta_k)$  is the transistor and diode conduction angle during the switching interval (event k), and  $\theta_s (= \omega_s t)$  is represented by the switching frequency of converter.

The steady-state solution of discrete state equation for output variable  $i_o$  is obtained by substituting steady-state condition into (A.21) as

$$\begin{aligned} \bar{I}_o &= i_o|_{(\beta_k=\bar{\beta}, \alpha_k=\bar{\alpha}, \gamma_k=\bar{\gamma}, I_{r,0(k)}=\bar{I}_r, V_{Cr,0(k)}=\bar{V}_{Cr}, V_{o,0(k)}=\bar{V}_o, V_{g,0(k)}=\bar{V}_g)} \end{aligned} \quad (\text{A.23})$$

*Step 5: Define State Variable.* Since the discrete large-signal state equations in (A.15), (A.16), and (A.21) have a high nonlinearity, control design technique based on the linear control theory cannot directly be applied. To obtain a linear state-space model, therefore, the linearization of large-signal equation is necessary. Equation (A.24) gives the definitions of interesting state variables in both the  $k_{th}$  switching event ( $t_{0(k)} \leq t \leq t_{2(k)}$ ) and the  $(k+1)_{th}$  switching event ( $t_{2(k)} \leq t \leq t_{2(k+1)}$ ). Finally, the equations of approximation of derivative in (A.24) and (A.25) are used to convert the discrete state-equation (large-signal model) into continuous time [10].

$$\begin{aligned} x_{1(k)} &= I_{r,0(k)}, \\ x_{2(k)} &= V_{Cr,0(k)} \end{aligned} \quad (\text{A.24})$$

$$\begin{aligned} I_{r,2(k)} &= -x_{1(k+1)}, \\ V_{Cr,2(k)} &= -x_{2(k+1)}, \\ \dot{x}_i(t_k) &= \frac{x_{i(k+1)} - x_{i(k)}}{t_{0(k+1)} - t_{0(k)}} = \frac{\omega_s}{\gamma_k} (x_{i(k+1)} - x_{i(k)}) \end{aligned} \quad (\text{A.25})$$

where

$$\gamma_k = \omega_s (t_{2(k)} - t_{0(k)}) = \omega_s (t_{0(k+1)} - t_{0(k)}) \quad (\text{A.26})$$

By replacing the state variables in (A.15) and (A.16) with the defined state variables in (A.24) and applying the approximation of (A.25) for derivative, the nonlinear state-space model is given by

$$\begin{aligned} \dot{x}_{1(k)} &= \frac{\omega_s}{\gamma_k} \cdot [\sin(\omega_{rs}\beta_K) \cdot \sin(\omega_{rs}\alpha_K) - 1] \cdot x_{1(k)} \\ &\quad + \frac{\omega_s}{\gamma_k} \cdot \left[ \frac{1}{Z_r} \cdot \cos(\omega_{rs}\beta_K) \cdot \sin(\omega_{rs}\alpha_K) \right] \cdot x_{2(k)} \\ &\quad + \frac{\omega_s}{\gamma_k} \cdot \left[ \frac{-2}{Z_r} \cdot \sin(\omega_{rs}\alpha_K) + \frac{1}{Z_r} \cdot \cos(\omega_{rs}\beta_K) \right. \\ &\quad \cdot \sin(\omega_{rs}\alpha_K) \Big] \cdot V_{o,0(k)} + \frac{\omega_s}{\gamma_k} \cdot \left[ \frac{1}{Z_r} \cdot \sin(\omega_{rs}\alpha_K) \right. \\ &\quad \left. + \frac{-1}{Z_r} \cdot \cos(\omega_{rs}\beta_K) \cdot \sin(\omega_{rs}\alpha_K) \right] \cdot V_{g,0(k)} \\ &= f_1 \{x_{1(k)}, x_{2(k)}, V_{o,0(k)}, V_{g,0(k)}, \alpha_k\} \\ &= f_1 \{x_1, x_2, v_o, v_g, \alpha\} = \frac{\omega_s}{\gamma_k} \\ &\quad \cdot f_1^* \{x_1, x_2, v_o, v_g, \alpha\} \end{aligned} \quad (\text{A.27})$$



$$\begin{aligned}
\dot{x}_{2(k)} &= \frac{\omega_s}{\gamma_k} \cdot [-Z_r \cdot \sin(\omega_{rs}\beta_K) \cdot \cos(\omega_{rs}\alpha_K)] \\
&\cdot x_{1(k)} + \frac{\omega_s}{\gamma_k} \cdot [-\cos(\omega_{rs}\beta_K) \cdot \cos(\omega_{rs}\alpha_K) - 1] \\
&\cdot x_{2(k)} + \frac{\omega_s}{\gamma_k} \cdot [2 \cdot \cos(\omega_{rs}\alpha_K) - \cos(\omega_{rs}\beta_K) \\
&\cdot \cos(\omega_{rs}\alpha_K) - 1] \cdot V_{o,0(k)} + \frac{\omega_s}{\gamma_k} \cdot [-\cos(\omega_{rs}\alpha_K) \\
&+ \cos(\omega_{rs}\beta_K) \cdot \cos(\omega_{rs}\alpha_K)] \cdot V_{g,0(k)} \\
&= f_2 \{x_{1(k)}, x_{2(k)}, V_{o,0(k)}, V_{g,0(k)}, \alpha_k\} \\
&= f_2 \{x_1, x_2, v_o, v_g, \alpha\} = \frac{\omega_s}{\gamma_k} \\
&\cdot f_2^* \{x_1, x_2, v_o, v_g, \alpha\}
\end{aligned} \quad (A.28)$$

where the output equation is defined as

$$\begin{aligned}
i_o &= f_{out} \{x_{1(k)}, x_{2(k)}, V_{o,0(k)}, V_{g,0(k)}, \alpha_k\} \\
&= f_{out} \{x_1, x_2, v_o, v_g, \alpha\} \\
&= \frac{1}{\gamma_k} \cdot f_{out}^* \{x_1, x_2, v_o, v_g, \alpha\}
\end{aligned} \quad (A.29)$$

*Step 6: Linearization and Small-Signal Model.* Consider that all the interesting state variables in pervious steps are in the steady-state (near the certain operating point, *OP*) with a small perturbation; therefore, the nonlinear state equations can be formalized with *Taylor Series Expansion* in terms of the operating point (*OP*) and the perturbations:

(i) Resonant inductor current:

$$\begin{aligned}
[\bar{x}_1 + \tilde{x}_1] &= f_1 \{\bar{x}_1 + \tilde{x}_1, \bar{x}_2 + \tilde{x}_2, \bar{V}_o + \tilde{V}_o, \bar{V}_g \\
&+ \tilde{V}_g, \bar{\alpha} + \tilde{\alpha}\} = \frac{\omega_s}{\gamma_k} \cdot f_1^* \{\bar{x}_1 + \tilde{x}_1, \bar{x}_2 + \tilde{x}_2, \bar{V}_o \\
&+ \tilde{V}_o, \bar{V}_g + \tilde{V}_g, \bar{\alpha} + \tilde{\alpha}\}
\end{aligned} \quad (A.30)$$

where

$$\begin{aligned}
x_1 &= \bar{x}_1 + \tilde{x}_1, \\
x_2 &= \bar{x}_2 + \tilde{x}_2, \\
v_g &= \bar{V}_g + \tilde{V}_g, \\
v_o &= \bar{V}_o + \tilde{V}_o, \\
\alpha &= \bar{\alpha} + \tilde{\alpha}
\end{aligned} \quad (A.31)$$

and then

$$\begin{aligned}
[\bar{x}_1 + \tilde{x}_1] &= f_1 \{\bar{x}_1, \bar{x}_2, \bar{V}_o, \bar{V}_g, \bar{\alpha}\} + \frac{\partial f_1}{\partial x_1} \bigg|_{OP} \tilde{x}_1 \\
&+ \frac{\partial f_1}{\partial x_2} \bigg|_{OP} \tilde{x}_2 + \frac{\partial f_1}{\partial v_g} \bigg|_{OP} \tilde{V}_g
\end{aligned}$$

$$\begin{aligned}
&+ \frac{\partial f_1}{\partial v_o} \bigg|_{OP} \tilde{V}_o + \frac{\partial f_1}{\partial \alpha} \bigg|_{OP} \tilde{\alpha} \\
&+ \frac{1}{2!} \frac{\partial^2 f_1}{\partial x_1^2} \bigg|_{OP} \tilde{x}_1^2 + \dots
\end{aligned} \quad (A.32)$$

where the subscript *OP* indicates the steady-state point, where the derivatives are evaluated at that point.

$$OP = \bar{I}_r, \bar{V}_{Cr}, \bar{V}_o, \bar{V}_g, \bar{\alpha}, \quad (A.33)$$

$$f_1 \{\bar{x}_1, \bar{x}_2, \bar{V}_o, \bar{V}_g, \bar{\alpha}\} = \bar{I}_r = 0$$

(ii) Resonant capacitor voltage:

$$\begin{aligned}
[\bar{x}_2 + \tilde{x}_2] &= f_2 \{\bar{x}_1 + \tilde{x}_1, \bar{x}_2 + \tilde{x}_2, \bar{V}_o + \tilde{V}_o, \bar{V}_g \\
&+ \tilde{V}_g, \bar{\alpha} + \tilde{\alpha}\} = \frac{\omega_s}{\gamma_k} \cdot f_2^* \{\bar{x}_1 + \tilde{x}_1, \bar{x}_2 + \tilde{x}_2, \bar{V}_o \\
&+ \tilde{V}_o, \bar{V}_g + \tilde{V}_g, \bar{\alpha} + \tilde{\alpha}\}
\end{aligned} \quad (A.34)$$

where

$$\begin{aligned}
x_1 &= \bar{x}_1 + \tilde{x}_1, \\
x_2 &= \bar{x}_2 + \tilde{x}_2, \\
v_g &= \bar{V}_g + \tilde{V}_g, \\
v_o &= \bar{V}_o + \tilde{V}_o, \\
\alpha &= \bar{\alpha} + \tilde{\alpha}
\end{aligned} \quad (A.35)$$

and then

$$\begin{aligned}
[\bar{x}_2 + \tilde{x}_2] &= f_2 \{\bar{x}_1, \bar{x}_2, \bar{V}_o, \bar{V}_g, \bar{\alpha}\} + \frac{\partial f_2}{\partial x_1} \bigg|_{OP} \tilde{x}_1 \\
&+ \frac{\partial f_2}{\partial x_2} \bigg|_{OP} \tilde{x}_2 + \frac{\partial f_2}{\partial v_g} \bigg|_{OP} \tilde{V}_g \\
&+ \frac{\partial f_2}{\partial v_o} \bigg|_{OP} \tilde{V}_o + \frac{\partial f_2}{\partial \alpha} \bigg|_{OP} \tilde{\alpha} \\
&+ \frac{1}{2!} \frac{\partial^2 f_2}{\partial x_1^2} \bigg|_{OP} \tilde{x}_1^2 + \dots
\end{aligned} \quad (A.36)$$

where

$$f_2 \{\bar{x}_1, \bar{x}_2, \bar{V}_o, \bar{V}_g, \bar{\alpha}\} = \bar{V}_{Cr} = 0 \quad (A.37)$$

(iii) Output current equation:

$$\begin{aligned}
\bar{I}_o + \tilde{I}_o &= f_{out} \{\bar{x}_1 + \tilde{x}_1, \bar{x}_2 + \tilde{x}_2, \bar{V}_o + \tilde{V}_o, \bar{V}_g \\
&+ \tilde{V}_g, \bar{\alpha} + \tilde{\alpha}\} = \frac{1}{(\bar{\gamma} + \tilde{\gamma})} \cdot f_{out}^* \{\bar{x}_1 + \tilde{x}_1, \bar{x}_2 \\
&+ \tilde{x}_2, \bar{V}_o + \tilde{V}_o, \bar{V}_g + \tilde{V}_g, \bar{\alpha} + \tilde{\alpha}\}
\end{aligned} \quad (A.38)$$

where

$$\begin{aligned}
 x_1 &= \bar{x}_1 + \tilde{x}_1, \\
 x_2 &= \bar{x}_2 + \tilde{x}_2, \\
 v_g &= \bar{V}_g + \tilde{V}_g, \\
 v_o &= \bar{V}_o + \tilde{V}_o, \\
 \alpha &= \bar{\alpha} + \tilde{\alpha}, \\
 \tilde{\gamma} &= \tilde{\alpha} + \tilde{\beta}
 \end{aligned} \tag{A.39}$$

and then

$$\begin{aligned}
 \bar{I}_o + \tilde{I}_o &= f_{out} \{ \bar{x}_1, \bar{x}_2, \bar{V}_o, \bar{V}_g, \bar{\alpha} \} + \left. \frac{\partial f_{out}}{\partial x_1} \right|_{OP} \tilde{x}_1 \\
 &+ \left. \frac{\partial f_{out}}{\partial x_2} \right|_{OP} \tilde{x}_2 + \left. \frac{\partial f_{out}}{\partial v_g} \right|_{OP} \tilde{V}_g + \left. \frac{\partial f_{out}}{\partial v_o} \right|_{OP} \tilde{V}_o \\
 &+ \left. \frac{\partial f_{out}}{\partial \alpha} \right|_{OP} \tilde{\alpha} + \frac{1}{2!} \left. \frac{\partial^2 f_{out}}{\partial x_1^2} \right|_{OP} \tilde{x}_1^2 + \dots = \frac{1}{\tilde{\gamma}} \cdot \left\{ 1 \right. \\
 &- \left( \frac{\tilde{\gamma}}{\gamma} \right) + \left( \frac{\tilde{\gamma}}{\gamma} \right)^2 - \dots + \dots \left. \right\} \\
 &\cdot \left\{ f_{out}^* \{ \bar{x}_1, \bar{x}_2, \bar{V}_o, \bar{V}_g, \bar{\alpha} \} + \left. \frac{\partial f_{out}^*}{\partial x_1} \right|_{OP} \tilde{x}_1 \right. \\
 &+ \left. \left. \left. \left. \left. \frac{\partial f_{out}^*}{\partial x_2} \right|_{OP} \tilde{x}_2 + \frac{\partial f_{out}^*}{\partial v_g} \right|_{OP} \tilde{V}_g + \frac{\partial f_{out}^*}{\partial v_o} \right|_{OP} \tilde{V}_o \right. \right. \right.
 \end{aligned}$$

$$+ \left. \frac{\partial f_{out}^*}{\partial \alpha} \right|_{OP} \tilde{\alpha} + \frac{1}{2!} \left. \frac{\partial^2 f_{out}^*}{\partial x_1^2} \right|_{OP} \tilde{x}_1^2 + \dots \left. \right\} \tag{A.40}$$

where

$$\begin{aligned}
 \bar{I}_o &= f_{out} \{ \bar{x}_1, \bar{x}_2, \bar{V}_o, \bar{V}_g, \bar{\alpha} \} \\
 \tilde{I}_o &= \frac{1}{\tilde{\gamma}} \cdot \left\{ \left[ \left. \frac{\partial f_{out}^*}{\partial x_1} \right|_{OP} - \bar{I}_o \cdot \left. \frac{\partial \beta}{\partial x_1} \right|_{OP} \right] \cdot \tilde{x}_1 \right. \\
 &+ \left[ \left. \frac{\partial f_{out}^*}{\partial x_2} \right|_{OP} - \bar{I}_o \cdot \left. \frac{\partial \beta}{\partial x_2} \right|_{OP} \right] \cdot \tilde{x}_2 \\
 &+ \left[ \left. \frac{\partial f_{out}^*}{\partial v_g} \right|_{OP} - \bar{I}_o \cdot \left. \frac{\partial \beta}{\partial v_g} \right|_{OP} \right] \cdot \tilde{V}_g \\
 &+ \left[ \left. \frac{\partial f_{out}^*}{\partial v_o} \right|_{OP} - \bar{I}_o \cdot \left. \frac{\partial \beta}{\partial v_o} \right|_{OP} \right] \cdot \tilde{V}_o \\
 &+ \left. \left[ \left. \frac{\partial f_{out}^*}{\partial \alpha} \right|_{OP} - \bar{I}_o \right] \cdot \tilde{\alpha} \right\}
 \end{aligned} \tag{A.41}$$

Neglect the higher-order terms of perturbation signals and retain only the linear terms in *Taylor Series Expansion* to obtain the linearized equations for (A.32), (A.36), and (A.40).

*Step 7: State-Space Model.* Equation (A.42) gives a linearized state-space model of SRC in subresonant mode from (A.32), (A.36), and (A.40) and the transfer functions between input state variables and the defined states are summarized in (A.50) and (A.52).

$$\begin{aligned}
 \begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} &= \begin{bmatrix} \left. \frac{\partial f_1}{\partial x_1} \right|_{OP} & \left. \frac{\partial f_1}{\partial x_2} \right|_{OP} \\ \left. \frac{\partial f_2}{\partial x_1} \right|_{OP} & \left. \frac{\partial f_2}{\partial x_2} \right|_{OP} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} \left. \frac{\partial f_1}{\partial \alpha} \right|_{OP} & \left. \frac{\partial f_1}{\partial v_g} \right|_{OP} & \left. \frac{\partial f_1}{\partial v_o} \right|_{OP} \\ \left. \frac{\partial f_2}{\partial \alpha} \right|_{OP} & \left. \frac{\partial f_2}{\partial v_g} \right|_{OP} & \left. \frac{\partial f_2}{\partial v_o} \right|_{OP} \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{V}_g \\ \tilde{V}_o \end{bmatrix} \\
 \tilde{I}_o &= \left[ \frac{1}{\tilde{\gamma}} \cdot \left[ \left. \frac{\partial f_{out}^*}{\partial x_1} \right|_{OP} - \bar{I}_o \cdot \left. \frac{\partial \beta}{\partial x_1} \right|_{OP} \right] \right] \frac{1}{\tilde{\gamma}} \cdot \left[ \left. \frac{\partial f_{out}^*}{\partial x_2} \right|_{OP} - \bar{I}_o \cdot \left. \frac{\partial \beta}{\partial x_2} \right|_{OP} \right] \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} \\
 &+ \left[ \frac{1}{\tilde{\gamma}} \cdot \left[ \left. \frac{\partial f_{out}^*}{\partial \alpha} \right|_{OP} - \bar{I}_o \right] \right] \frac{1}{\tilde{\gamma}} \cdot \left[ \left. \frac{\partial f_{out}^*}{\partial v_g} \right|_{OP} - \bar{I}_o \cdot \left. \frac{\partial \beta}{\partial v_g} \right|_{OP} \right] \frac{1}{\tilde{\gamma}} \cdot \left[ \left. \frac{\partial f_{out}^*}{\partial v_o} \right|_{OP} - \bar{I}_o \cdot \left. \frac{\partial \beta}{\partial v_o} \right|_{OP} \right] \begin{bmatrix} \tilde{\alpha} \\ \tilde{V}_g \\ \tilde{V}_o \end{bmatrix}
 \end{aligned} \tag{A.42}$$

For derivative of equations  $f_1$  and  $f_2$ ,

$$\frac{\partial f_i}{\partial x_j} = \left( \frac{\partial}{\partial x_j} \frac{\omega_s}{\gamma} \right) \cdot f_i^* \Big|_{OP} + \frac{\omega_s}{\gamma} \frac{\partial f_i^*}{\partial x_j} \Big|_{OP} \tag{A.43}$$

where

$$\gamma = \alpha + \beta \quad \text{for } i = 1, 2, \quad j = 1, 2 \tag{A.44}$$

With the steady-state operating conditions,

$$\begin{aligned}
 \frac{\omega_s}{\gamma} \Big|_{OP} &\neq 0, \\
 f_i^* \Big|_{OP} &= 0
 \end{aligned} \tag{A.45}$$

Therefore,

$$\frac{\partial f_i}{\partial x_j} = \frac{\omega_s}{\gamma} \frac{\partial f_i^*}{\partial x_j} \Big|_{OP} \quad (\text{A.46})$$

The same approach as the derivative of  $f_1$  and  $f_2$  with respect to input states  $x_1$  and  $x_2$  can be used to evaluate the derivative of  $f_{out}^*$  and the derivative of  $f_1$  and  $f_2$  with respect to input states  $\alpha$ ,  $v_g$ , and  $v_o$ .

According to the derivation of large signal model in (A.8), the angle  $\beta$  and its steady-state solution can be expressed by

$$\tan(\omega_{rs}\beta) = \frac{-x_1 Z_r}{(v_g - v_o - x_2)} = \tan(\omega_{rs}\beta - \pi), \quad (\text{A.47})$$

$$\beta|_{OP} = \frac{\pi}{\omega_{rs}} + \frac{1}{\omega_{rs}} \cdot \tan^{-1} \left[ \frac{-\bar{I}_r Z_r}{(\bar{V}_g - \bar{V}_o - \bar{V}_{Cr})} \right] \quad (\text{A.48})$$

The derivatives of  $\beta$  with respect to input states  $x_1$ ,  $x_2$ ,  $v_g$ , and  $v_o$  at the given operating points are

$$\begin{aligned} \frac{\partial \beta}{\partial x_1} \Big|_{OP} &= \left[ \frac{1}{\omega_{rs}} \cdot \frac{-Z_r \cdot (v_g - v_o - x_2)}{(v_g - v_o - x_2)^2 + (x_1 Z_r)^2} \right] \Big|_{OP} \\ &= \frac{1}{\omega_{rs}} \cdot \frac{-Z_r \cdot (\bar{V}_g - \bar{V}_o - \bar{V}_{Cr})}{(\bar{V}_g - \bar{V}_o - \bar{V}_{Cr})^2 + (\bar{I}_r Z_r)^2} \\ \frac{\partial \beta}{\partial x_2} \Big|_{OP} &= \left[ \frac{1}{\omega_{rs}} \cdot \frac{-x_1 Z_r}{(v_g - v_o - x_2)^2 + (x_1 Z_r)^2} \right] \Big|_{OP} \\ &= \frac{1}{\omega_{rs}} \cdot \frac{-\bar{I}_r Z_r}{(\bar{V}_g - \bar{V}_o - \bar{V}_{Cr})^2 + (\bar{I}_r Z_r)^2} \\ \frac{\partial \beta}{\partial v_o} \Big|_{OP} &= \left[ \frac{1}{\omega_{rs}} \cdot \frac{-x_1 Z_r}{(v_g - v_o - x_2)^2 + (x_1 Z_r)^2} \right] \Big|_{OP} \\ &= \frac{1}{\omega_{rs}} \cdot \frac{-\bar{I}_r Z_r}{(\bar{V}_g - \bar{V}_o - \bar{V}_{Cr})^2 + (\bar{I}_r Z_r)^2} \\ \frac{\partial \beta}{\partial v_g} \Big|_{OP} &= \left[ \frac{1}{\omega_{rs}} \cdot \frac{x_1 Z_r}{(v_g - v_o - x_2)^2 + (x_1 Z_r)^2} \right] \Big|_{OP} \\ &= \frac{1}{\omega_{rs}} \cdot \frac{\bar{I}_r Z_r}{(\bar{V}_g - \bar{V}_o - \bar{V}_{Cr})^2 + (\bar{I}_r Z_r)^2} \end{aligned} \quad (\text{A.49})$$

where  $\omega_{rs} (= \omega_r / \omega_s)$  is defined as the ratio between the natural frequency ( $\omega_r$ ) of resonant tank and the switching frequency of converter ( $\omega_s$ ).

*Step 8: Transfer Function.* The transfer functions between the converter output current (output rectifier current) and input state variables can be obtained with (A.42):

$$\tilde{I}_o(s) = \begin{bmatrix} g_1(s) & g_2(s) & g_3(s) \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{V}_g \\ \tilde{V}_o \end{bmatrix} \quad (\text{A.50})$$

where

$$\begin{aligned} g_1(s) &= \frac{\tilde{I}_o(s)}{\tilde{\alpha}(s)} \Big|_{\tilde{V}_g(s)=0, \tilde{V}_o(s)=0}, \\ g_2(s) &= \frac{\tilde{I}_o(s)}{\tilde{V}_g(s)} \Big|_{\tilde{\alpha}(s)=0, \tilde{V}_o(s)=0}, \\ g_3(s) &= \frac{\tilde{I}_o(s)}{\tilde{V}_o(s)} \Big|_{\tilde{\alpha}(s)=0, \tilde{V}_g(s)=0}, \\ \tilde{f}_s &= \tilde{\alpha}(s) \cdot \frac{f_r}{-\pi} \end{aligned} \quad (\text{A.51})$$

and transfer functions between defined internal state variables and input state are

$$\begin{aligned} \tilde{X} &= \begin{bmatrix} \tilde{I}_r \\ \tilde{v}_{cr} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} \\ &= \begin{bmatrix} g_{xu,11} & g_{xu,12} & g_{xu,13} \\ g_{xu,21} & g_{xu,22} & g_{xu,23} \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{V}_g \\ \tilde{V}_o \end{bmatrix} \end{aligned} \quad (\text{A.52})$$

where

$$\begin{aligned} g_{xu,11} &= \frac{\tilde{I}_r}{\tilde{\alpha}} \Big|_{\tilde{V}_g(s)=0, \tilde{V}_o(s)=0}, \\ g_{xu,12} &= \frac{\tilde{I}_r}{\tilde{V}_g} \Big|_{\tilde{\alpha}(s)=0, \tilde{V}_o(s)=0}, \\ g_{xu,13} &= \frac{\tilde{I}_r}{\tilde{V}_o} \Big|_{\tilde{\alpha}(s)=0, \tilde{V}_g(s)=0}, \\ g_{xu,21} &= \frac{\tilde{v}_{cr}}{\tilde{\alpha}} \Big|_{\tilde{V}_g(s)=0, \tilde{V}_o(s)=0}, \\ g_{xu,22} &= \frac{\tilde{v}_{cr}}{\tilde{V}_g} \Big|_{\tilde{\alpha}(s)=0, \tilde{V}_o(s)=0}, \\ g_{xu,23} &= \frac{\tilde{v}_{cr}}{\tilde{V}_o} \Big|_{\tilde{\alpha}(s)=0, \tilde{V}_g(s)=0} \end{aligned} \quad (\text{A.53})$$

The derivation of the linearized state-space model and the expression of elements in [A], [B], [C], and [D] matrix are given in (A.42). The transfer functions,  $g_1(s)$ ,  $g_2(s)$ , and  $g_3(s)$ , in (A.50) can be obtained by the formula of  $C(sI - A)^{-1}B + D$ .

## Nomenclature

$L_r$ :	Inductor in resonant tank
$C_r$ :	Capacitor in resonant tank
$i_r$ :	Resonant inductor current
$v_{Cr}$ :	Resonant capacitor voltage
$v_g$ :	Input voltage of resonant tank referred to as secondary side of medium-frequency transformer
$v_o$ :	Output voltage of resonant tank
$V_{LVDC}$ :	Low voltage DC
$V_{MVDC}$ :	Medium voltage DC
$i_{out,Rec}$ :	Output current of diode rectifier
$i_{turb}$ :	Output current of DC wind turbine converter
$L_f$ :	Inductor in output filter
$C_f$ :	Capacitor in output filter
$f_s$ :	Switching frequency of series resonant converter defined by $f_s = \omega_s/2\pi$
$\omega_r$ :	Natural resonant frequency of tank defined by $\omega_r = 1/\sqrt{L_r C_r}$
$\alpha_k$ :	Transistor and diode conduction angle during event $k$
$\beta_k$ :	Transistor conduction angle during event $k$
$\gamma_k$ :	Total duration of event ( $\gamma_k = \alpha_k + \beta_k$ ).

## Data Availability

The authors of the manuscript declare that the data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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